3.2.1 Example

There are several different ways to solve a linear optimisation problem, and many variables and constraints can be included. For example, take this simple linear optimisation problem over two variables.

$$\begin{aligned} \text{minimise} \quad & f(\mathbf{x}) = 2x_1^2 + x_2^2 - 5x_1 \\ \text{subject to} \quad & x_1 + x_2 \geq 3 \\ & 2x_2 - x_1 \geq -3 \\ & x_2 - 2x_1 \geq -8 \\ & x_1 \geq 0, x_2 \geq 0 \end{aligned} \tag{4}$$

For this two-dimensional example we can plot all of the inequalities and highlight the region that satisfies them all as can be seen in 1. This is the feasible region.

Questo grafico non va bene

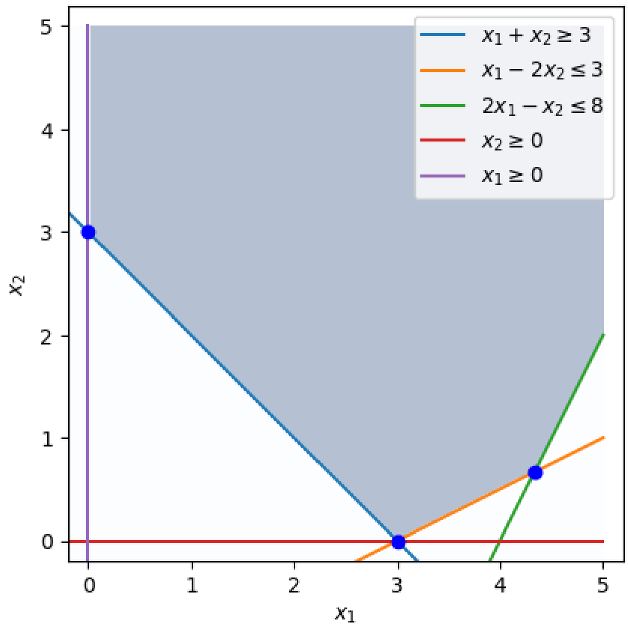

Figure 1: Feasible region of the linear programming problem

The lines that represent the constraints of the problem form vertices that lie on the edge of the feasible region where they intersect. It is at one of these points that we will find the optimal solution, this is done by evaluating the objective function $f(\mathbf{x})$, and finding the minimal value produced out of the vertices.

x_1	x_2	$f(\mathbf{x})$
0	3	9
3	0	3
13/3	2/3	16.33

Table 1: Feasible Solutions

Then the minimal value is $f(\mathbf{x}) = 3$, with the variables taking the value $\mathbf{x}^T = (3, 0)$.

This is an efficient way of solving a linear programming problem with a small number of constraints and variables. However, as the number of constraints and variables are increased, so does the complexity of the problem. in these cases, various computation algorithms are often used.

3.3 Multi-Objective Optimisation problem

Definition 3.2 (multi-objective optimisation problem)**.** is a mathematical optimisation problem, where several objective functions are considered and optimised at the same time.

For the purposes of this project we will only consider problems that minimise the objective functions.

Definition 3.3 (Standard Multi-Objectivive Minimisation Problem)**.** This problem is to find a vector $\mathbf{x} \in \mathbb{R}^n$